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饱和黏性土地基中现浇大直径管桩水平振动响应解析解

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摘 要：基于经典的平面应变假定，将土体假设为若干个相互独立的薄层，对饱和土地基中现浇大直径管桩水平振动频域特性进行了理论研究。首先通过引入势函数对土体动力固结方程解耦，采用 Laplace 变换和分离变量的方法求得了桩周土及桩芯土频域响应解析解，进而利用桩土完全耦合的条件得到桩振动响应解，给出了桩顶复阻抗解析表达式。将本文解完全退化到实心桩的解与已有理论解对比，验证了本文解的合理性并阐明两解的区别。通过参数分析，得到了桩长以及桩半径等参数对复阻抗特性影响的规律。

关键词：现浇大直径管桩；水平振动；解析解；复阻抗；饱和土

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Analytical solution of horizontal vibration of cast-in-place large-diameter pipe piles in saturated soils

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Abstract: Based on the classical plane strain assumption, considering the soils as a series of independent thin layers, the frequency-domain horizontal dynamic response of a cast-in-situ large-diameter concrete pipe pile in saturated soils is investigated. The potential functions are employed to decouple the consolidation equations of the soils, the analytical solutions of the outer and inner soils in frequency domain are obtained by the Laplace transform technique and the separation of variables. Under the assumption of perfect contacts between the pile and both the outer and inner soils, the response of the pile is derived and its complex impedances are obtained. The derived solution is compared with that of a solid pile to verify the rationality, and the difference between the two solutions is illustrated. The effects of the pile length and section radius on the complex impedance of pile top are analyzed.

Key words: PCC pile; horizontal vibration; analytical solution; complex impedance; saturated soil

0 引 言

桩基础在工程应用中经常会受到地震荷载、波浪荷载、上部结构风荷载等水平动荷载。水平动荷载下桩土相互作用问题近几十年来受到学者的广泛关注。很多学者基于Winkler模型理论对桩基动力特性进行了大量的研究[1-3]。但Winkler模型的缺点是不能真正的反应桩－土耦合振动，而且在实际工程中模型参数的确定常带有经验性,参数取值的正确与否直接影响着计算结果。Novak[4-5]假定土体由一系列相互独立的薄层组成，层与层之间相互独立，桩土之间位移和应力连续，推导了桩顶复阻抗的解析表达式，这就是经典的平面应变解。Novak的平面应变假定理论在后面学者的研究中应用广泛。Gazetas等[6]基于平面应变假定提出了一种辐射阻尼简化算法，研究了成层土中桩的水平振动问题。杨军等[7]将平面应变假定和Gazetas的分象限假设推广到饱和土中桩的水平振动问题。尚守平等[8]忽略土中水体的惯性项，通过引入势函数解耦土体Biot固结方程，研究了平面应变假定下饱和土体中单桩水平振动问题。

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现浇混凝土大直径管桩（以下简称PCC桩）是近年来河海大学岩土工程研究所自主开发研制的一种新的具有自主知识产权的地基处理技术，由于其施工工艺简便，大量节省混凝土并有效提高桩基承载力，已在软土地区取得广泛应用并开展了纵向振动问题的研究[9-15]。对于PCC桩水平振动问题，同时存在桩芯土和桩周土的作用，桩芯土与管桩如何作用，是一个值得研究的问题。本文基于平面应变假定和Biot动力固结理论，推导了任意动荷载作用下饱和土中PCC桩水平振动响应频域解析解，给出了桩顶复阻抗解析表达式，丰富了桩基振动理论，对PCC桩的工程实践应用也有一定的指导意义。

1 基本假定及方程的建立

1.1 计算简图及基本假定

计算简图如图1所示，桩长为 H，桩材密度为 ρ_{p}，桩的弹性模量为 E_{p}，桩截面外半径为 r_1，内半径为 r_2。

本文在建立方程时采用了以下假定：

（1）桩周土和桩芯土为一系列相互独立的薄层组成，忽略土层之间的相互作用。

（2）桩周土和桩芯土以及桩身混凝土为均匀弹性体。

（3）桩土系统振动为小变形，桩与桩周土、桩芯土完全连续接触，且桩土接触面不透水。

（4）桩土体系水平耦合振动时，忽略土体的纵向位移。

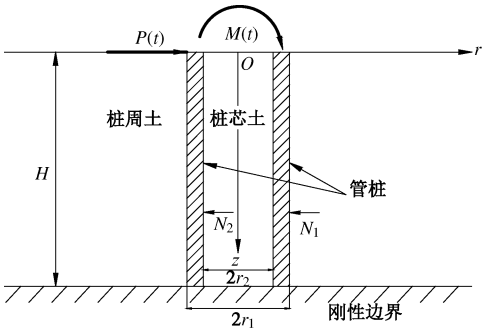


图 1 计算模型示意图

Fig. 1 Computational model

1.2 方程的建立

根据 Biot[16]动力固结理论，容易建立轴对称条件下桩周土的运动方程为

$$G_1\nabla^2 u_{r1}+(\lambda_1+G_1+\alpha_1^2M_1)\frac{\partial e_1}{\partial r}-\frac{G_1}{r^2}(2\frac{\partial u_{\theta1}}{\partial\theta}+u_{r1})-\alpha_1M_1\frac{\partial\zeta_1}{\partial r}=\rho_1\frac{\partial^2u_{r1}}{\partial t^2}+\rho_{\mathrm{f}}\frac{\partial^2w_{r1}}{\partial t^2}\quad,\tag{1}$$

$$G_1\nabla^2 u_{\theta1}+(\lambda_1+G_1+\alpha_1^2M_1)\frac{1}{r}\frac{\partial e_1}{\partial\theta}-\frac{G_1}{r^2}(u_{\theta1}-2\frac{\partial u_{r1}}{\partial\theta})-\alpha_1M_1\frac{1}{r}\frac{\partial\zeta_1}{\partial\theta}=\rho_1\frac{\partial^2u_{\theta1}}{\partial t^2}+\rho_{\mathrm{f}}\frac{\partial^2w_{\theta1}}{\partial t^2}\quad,\tag{2}$$

$$\alpha_1M_1\frac{\partial e_1}{\partial r}-M_1\frac{\partial\zeta_1}{\partial r}=\rho_{\mathrm{f}}\frac{\partial^2u_{r1}}{\partial t^2}+m_1\frac{\partial^2w_{r1}}{\partial t^2}+b_1\frac{\partial w_{r1}}{\partial t}\,,\tag{3}$$

$$\alpha_1M_1\frac{1}{r}\frac{\partial e_1}{\partial\theta}-M_1\frac{1}{r}\frac{\partial\zeta_1}{\partial\theta}=\rho_{\mathrm{f}}\frac{\partial^2u_{\theta1}}{\partial t^2}+m_1\frac{\partial^2w_{\theta1}}{\partial t^2}+b_1\frac{\partial w_{\theta1}}{\partial t}\,。\tag{4}$$

轴对称条件下桩芯土用位移表示的运动方程为

$$G_2\nabla^2 u_{r2}+(\lambda_2+G_2+\alpha_2^2M_2)\frac{\partial e_2}{\partial r}-\frac{G_2}{r^2}(2\frac{\partial u_{\theta2}}{\partial\theta}+u_{r2})-\alpha_2M_2\frac{\partial\zeta_2}{\partial r}=\rho_2\frac{\partial^2u_{r2}}{\partial t^2}+\rho_{\mathrm{f}}\frac{\partial^2w_{r2}}{\partial t^2}\quad,\tag{5}$$

$$G_2\nabla^2 u_{\theta2}+(\lambda_2+G_2+\alpha_2^2M_2)\frac{1}{r}\frac{\partial e_2}{\partial\theta}-\frac{G_2}{r^2}(u_{\theta2}-2\frac{\partial u_{r2}}{\partial\theta})-\alpha_2M_2\frac{1}{r}\frac{\partial\zeta_2}{\partial\theta}=\rho_2\frac{\partial^2u_{\theta2}}{\partial t^2}+\rho_{\mathrm{f}}\frac{\partial^2w_{\theta2}}{\partial t^2}\quad,\tag{6}$$

$$\alpha_2M_2\frac{\partial e_2}{\partial r}-M_2\frac{\partial\zeta_2}{\partial r}=\rho_{\mathrm{f}}\frac{\partial^2u_{r2}}{\partial t^2}+m_2\frac{\partial^2w_{r2}}{\partial t^2}+b_2\frac{\partial w_{r2}}{\partial t}\,,\tag{7}$$

$$\alpha_2M_2\frac{1}{r}\frac{\partial e_2}{\partial\theta}-M_2\frac{1}{r}\frac{\partial\zeta_2}{\partial\theta}=\rho_{\mathrm{f}}\frac{\partial^2u_{\theta2}}{\partial t^2}+m_2\frac{\partial^2w_{\theta2}}{\partial t^2}+b_2\frac{\partial w_{\theta2}}{\partial t}\,。\tag{8}$$

式中 $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$； $e_1=\frac{\partial u_{r1}}{\partial r}+\frac{u_{r1}}{r}+\frac{\partial u_{z1}}{\partial z}$； $\zeta_1=-(\frac{\partial w_{r1}}{\partial r}+\frac{w_{r1}}{r}+\frac{\partial w_{z1}}{\partial z})$； $e_2=\frac{\partial u_{r2}}{\partial r}+\frac{u_{r2}}{r}+\frac{\partial u_{z2}}{\partial z}$； $\zeta_2=-(\frac{\partial w_{r2}}{\partial r}+\frac{w_{r2}}{r}+\frac{\partial w_{z2}}{\partial z})$； u_{r1}， $u_{\theta1}$ 和 u_{r2}， $u_{\theta2}$ 分别为桩周土和桩芯土土体的水平向和环向位移， w_{r1}， $w_{\theta1}$ 和 w_{r2}， $w_{\theta2}$ 分别为桩周土和桩芯土流体介质相对于土体的水平向和环向位移， λ_1， G_1 和 λ_2， G_2 分别为桩周土和桩芯土的拉梅常数， ρ_1， ρ_2 分别为桩周土和桩芯土的密度， $\rho_1=(1-n_1)\rho_{\mathrm{s1}}+n_1\rho_{\mathrm{f}}$， $\rho_2=(1-n_2)\rho_{\mathrm{s2}}+n_2\rho_{\mathrm{f}}$， ρ_{s1}， ρ_{s2} 分别为桩周土和桩芯土土颗粒的密度； n_1， n_2 分别为桩周土和桩芯土的孔隙比； b_1， b_2 分别为桩周土和桩芯土的黏性耦合系数， $b_1=\rho_{\mathrm{f}}g/k_{\mathrm{d1}}$， $b_2=\rho_{\mathrm{f}}g/k_{\mathrm{d2}}$， ρ_{f} 为流体介质的密度， g 为重力加速度， k_{d1}， k_{d2} 分别为桩周土和桩芯土的渗透系数； m_1， m_2 分别为桩周土和桩芯土的密度系数， $m_1=\rho_{\mathrm{f}}/n_1$， $m_2=\rho_{\mathrm{f}}/n_2$； $\alpha_1=1-K_{\mathrm{b1}}/K_{\mathrm{s1}}$， $\alpha_2=1-K_{\mathrm{b1}}/K_{\mathrm{s1}}$， $1/M_1=(1-n_1)/K_{\mathrm{s1}}+n_1/K_{\mathrm{f}}$， $1/M_2=(1-n_2)/K_{\mathrm{s2}}+n_2/K_{\mathrm{f}}$， K_{s1}， K_{s2}， K_{f}， K_{b1} 以及 K_{b2} 分别为桩周土和桩芯土土颗粒、流体及图骨架的体积模量。

令 u_{p} 为桩身水平位移，取桩身微元体作动力平衡分析，可得桩水平振动基本方程：

$$E_{\mathrm{p}}I_{\mathrm{p}}\frac{\partial^4u_{\mathrm{p}}(z,t)}{\partial z^4}+m\frac{\partial^2u_{\mathrm{p}}(z,t)}{\partial t^2}+N_1+N_2=0\,,\tag{9}$$

式中， I_{p} 为桩的截面惯性矩， m 为桩单位长度的质量， N_1， N_2 分别为水平振动时轴向单位长度桩周土和桩芯土的横向作用力。

1.3　边界条件

桩底为固定边界：

$$\left.\begin{aligned} &u_{\mathrm{p}}\big|_{z=H}=0\ ,\\ &\frac{\partial u_{\mathrm{p}}}{\partial z}\Big|_{z=H}=0\ 。\end{aligned}\right\} \tag{10}$$

桩土接触面不透水：

$$w_{r1}\big|_{r=r_1}=0\quad , \tag{11}$$

$$w_{r2}\big|_{r=r_2}=0\quad 。 \tag{12}$$

桩土接触面处土体位移连续条件为

$$u_{r1}\big|_{r=r_1}=u_{\mathrm{p}}\cos\theta\quad , \tag{13}$$

$$u_{\theta1}\big|_{r=r_1}=-u_{\mathrm{p}}\sin\theta\quad , \tag{14}$$

$$u_{r2}\big|_{r=r_2}=u_{\mathrm{p}}\cos\theta\quad , \tag{15}$$

$$u_{\theta2}\big|_{r=r_2}=-u_{\mathrm{p}}\sin\theta\quad 。 \tag{16}$$

分析中假定初始状态为静止。

2　方程的求解

2.1　桩周土振动方程的求解

引入势函数 $\phi_{\mathrm{s1}}(r,\theta,t)$，$\phi_{\mathrm{f1}}(r,\theta,t)$，$\psi_{\mathrm{s1}}(r,\theta,t)$ 和 $\psi_{\mathrm{f1}}(r,\theta,t)$，使得

$$\left.\begin{aligned} u_{r1}&=\frac{\partial\phi_{\mathrm{s1}}}{\partial r}+\frac{1}{r}\frac{\partial\psi_{\mathrm{s1}}}{\partial\theta}\ ,\\ u_{\theta1}&=\frac{1}{r}\frac{\partial\phi_{\mathrm{s1}}}{\partial\theta}-\frac{\partial\psi_{\mathrm{s1}}}{\partial r}\ ,\\ w_{r1}&=\frac{\partial\phi_{\mathrm{f1}}}{\partial r}+\frac{1}{r}\frac{\partial\psi_{\mathrm{f1}}}{\partial\theta}\ ,\\ w_{\theta1}&=\frac{1}{r}\frac{\partial\phi_{\mathrm{f1}}}{\partial\theta}-\frac{\partial\psi_{\mathrm{f1}}}{\partial r}\ 。\end{aligned}\right\} \tag{17}$$

将势函数代入式（1）～（4），并作 Laplace 变换，可得

$$(\lambda_1+2G_1+\alpha_1^2M_1)\nabla^2\Phi_{\mathrm{s1}}+\alpha_1M_1\nabla^2\Phi_{\mathrm{f1}}=\rho_1s^2\Phi_{\mathrm{s1}}+\rho_{\mathrm{f}}s^2\Phi_{\mathrm{f1}}, \tag{18}$$

$$G_1\nabla^2\Psi_{\mathrm{s1}}=\rho_1s^2\Psi_{\mathrm{s1}}+\rho_{\mathrm{f}}s^2\Psi_{\mathrm{f1}}\quad , \tag{19}$$

$$\alpha_1M_1\nabla^2\Phi_{\mathrm{s1}}+M_1\nabla^2\Phi_{\mathrm{f1}}=\rho_{\mathrm{f}}s^2\Phi_{\mathrm{s1}}+m_1s^2\Phi_{\mathrm{f1}}+b_1s\Phi_{\mathrm{f1}}\ , \tag{20}$$

$$\rho_{\mathrm{f}}s^2\Psi_{\mathrm{s1}}+m_1s^2\Psi_{\mathrm{f1}}+b_1s\Psi_{\mathrm{f1}}=0\quad , \tag{21}$$

式中，$\Phi_{\mathrm{s1}}(r,\theta,s)$，$\Phi_{\mathrm{f1}}(r,\theta,s)$，$\Psi_{\mathrm{s1}}(r,\theta,s)$ 和 $\Psi_{\mathrm{f1}}(r,\theta,s)$ 分别是 $\phi_{\mathrm{s1}}(r,\theta,t)$，$\phi_{\mathrm{f1}}(r,\theta,t)$，$\psi_{\mathrm{s1}}(r,\theta,t)$ 和 $\psi_{\mathrm{f1}}(r,\theta,t)$ 的 Laplace 变换。

表示成矩阵形式有

$$\begin{bmatrix}(\lambda_1+2G_1+\alpha_1^2M_1)\nabla^2-\rho_1s^2 & \alpha_1M_1\nabla^2-\rho_{\mathrm{f}}s^2\\ \alpha_1M_1\nabla^2-\rho_{\mathrm{f}}s^2 & M_1\nabla^2-m_1s^2-b_1s\end{bmatrix}\begin{Bmatrix}\Phi_{\mathrm{s1}}\\ \Phi_{\mathrm{f1}}\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}, \tag{22}$$

$$\begin{bmatrix}G_1\nabla^2-\rho_1s^2 & -\rho_{\mathrm{f}}s^2\\ \rho_{\mathrm{f}}s^2 & m_1s^2+b_1s\end{bmatrix}\begin{Bmatrix}\Psi_{\mathrm{s1}}\\ \Psi_{\mathrm{f1}}\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}\quad 。 \tag{23}$$

要使微分算子方程有非零解，微分算子行列式必须为零，可得

$$(\nabla^4-h_{11}\nabla^2+h_{12})\Phi_{\mathrm{s1}}=0\quad , \tag{24}$$

$$(\nabla^4-h_{11}\nabla^2+h_{12})\Phi_{\mathrm{f1}}=0\quad , \tag{25}$$

式中，

$$h_{11}=\frac{(\lambda_1+2G_1+\alpha_1^2M_1)(m_1s^2+b_1s)+\rho_1s^2M_1-2\alpha_1M_1\rho_{\mathrm{f}}s^2}{(\lambda_1+2G_1)M_1},$$

$$h_{12}=\frac{\rho_1s^2(m_1s^2+b_1s)-\rho_{\mathrm{f}}^2s^4}{(\lambda_1+2G_1)M_1}\ 。$$

式（18）可化为

$$(\nabla^2-\beta_{11}^2)(\nabla^2-\beta_{12}^2)\Phi_{\mathrm{s1}}=0\quad , \tag{26}$$

式中，$\beta_{11}^2=\dfrac{h_{11}+\sqrt{h_{11}^2-4h_{12}}}{2}$，$\beta_{12}^2=\dfrac{h_{11}-\sqrt{h_{11}^2-4h_{12}}}{2}$。

由算子分解理论[17]有：$\Phi_{\mathrm{s1}}=\Phi_{\mathrm{s11}}+\Phi_{\mathrm{s12}}$，其中 Φ_{s11} 和 Φ_{s12} 分别满足下式：

$$\left.\begin{aligned}(\nabla^2-\beta_{11}^2)\Phi_{\mathrm{s11}}=0\ ,\\ (\nabla^2-\beta_{12}^2)\Phi_{\mathrm{s12}}=0\ 。\end{aligned}\right\} \tag{27}$$

通过分离变量法求解可得

$$\Phi_{\mathrm{s11}}=[A_{11}\mathrm{K}_{m_1}(\beta_{11}r)+B_{11}\mathrm{I}_{m_1}(\beta_{11}r)](C_{11}\sin m_1\theta+D_{11}\cos m_1\theta)\quad , \tag{28}$$

$$\Phi_{\mathrm{s12}}=[A_{12}\mathrm{K}_{m_1}(\beta_{12}r)+B_{12}\mathrm{I}_{m_1}(\beta_{12}r)](C_{12}\sin m_1\theta+D_{12}\cos m_1\theta)\quad , \tag{29}$$

式中，$\mathrm{K}_{m_1}(\beta_{11}r)$，$\mathrm{K}_{m_1}(\beta_{12}r)$ 和 $\mathrm{I}_{m_1}(\beta_{11}r)$，$\mathrm{I}_{m_1}(\beta_{12}r)$ 分别为 m_1 阶第一类、第二类变形 Bessel 函数。

桩周土无穷远处位移为零，可得 $B_{11}=B_{12}=0$。u_{r1} 为 θ 的偶函数，$u_{\theta1}$ 为 θ 的奇函数，因此 $C_{11}=C_{12}=0$，$m_1=1$。

势函数 Φ_{s1} 可表示为

$$\Phi_{\mathrm{s1}}=[A_{11}\mathrm{K}_1(\beta_{11}r)+A_{12}\mathrm{K}_1(\beta_{12}r)]\cos\theta\quad , \tag{30}$$

同理可有

$$\Phi_{\mathrm{f1}}=[A_{13}\mathrm{K}_1(\beta_{11}r)+A_{14}\mathrm{K}_1(\beta_{12}r)]\cos\theta\quad , \tag{31}$$

$$\Psi_{\mathrm{s1}}=A_{15}\mathrm{K}_1(\beta_{13}r)\sin\theta\quad , \tag{32}$$

$$\Psi_{\mathrm{f1}}=A_{16}\mathrm{K}_1(\beta_{13}r)\sin\theta\quad , \tag{33}$$

式中，$\beta_{13}^2=\dfrac{\rho_1s^2(m_1s^2+b_1s)-\rho_{\mathrm{f}}^2s^4}{G_1(m_1s^2+b_1s)}$。

将式（30）～（33）代入式（22）、（23）中，可得到如下相关关系：

$$\left.\begin{aligned}A_{13}&=\chi_{11}A_{11}\ ,\\ A_{14}&=\chi_{12}A_{12}\ ,\\ A_{16}&=\chi_{13}A_{15}\ ,\end{aligned}\right\} \tag{34}$$

式中，$\chi_{11}=\dfrac{-\alpha_1M_1\beta_{11}^2+\rho_{\mathrm{f}}s^2}{M_1\beta_{11}^2-(m_1s^2+b_1s)}$，$\chi_{12}=\dfrac{-\alpha_1M_1\beta_{12}^2+\rho_{\mathrm{f}}s^2}{M_1\beta_{12}^2-(m_1s^2+b_1s)}$，$\chi_{13}=\dfrac{-\rho_{\mathrm{f}}s^2}{m_1s^2+b_1s}$。

桩周土土体和流体位移可表示为

$$U_{r1}=\left\{A_{11}\left[\mathrm{K}_1(\beta_{11}r)\right]'+A_{12}\left[\mathrm{K}_1(\beta_{12}r)\right]'+A_{15}\frac{1}{r}\mathrm{K}_1(\beta_{13}r)\right\}\cos\theta, \tag{35}$$

$$W_{r1}=\left\{\chi_{11}A_{11}\left[\mathrm{K}_1(\beta_{11}r)\right]'+\chi_{12}A_{12}\left[\mathrm{K}_1(\beta_{12}r)\right]'+\chi_{13}A_{15}\frac{1}{r}\mathrm{K}_1(\beta_{13}r)\right\}\cos\theta, \tag{36}$$

$$U_{\theta1}=\left\{-A_{11}\frac{1}{r}\mathrm{K}_1(\beta_{11}r)-A_{12}\frac{1}{r}\mathrm{K}_1(\beta_{12}r)-A_{15}\left[\mathrm{K}_1(\beta_{13}r)\right]'\right\}\sin\theta, \tag{37}$$

$$W_{\theta1}=\left\{-\chi_{11}A_{11}\frac{1}{r}\mathrm{K}_1(\beta_{11}r)-\chi_{12}A_{12}\frac{1}{r}\mathrm{K}_1(\beta_{12}r)-\chi_{13}A_{15}\left[\mathrm{K}_1(\beta_{13}r)\right]'\right\}\sin\theta。 \tag{38}$$

由桩周土与桩接触面边界条件，可求得

$$A_{11}=\frac{\gamma_{15}\gamma_{19}+\gamma_{15}\gamma_{13}-\gamma_{12}\gamma_{16}-\gamma_{16}\gamma_{18}}{\gamma_{14}\gamma_{18}\gamma_{13}-\gamma_{14}\gamma_{12}\gamma_{19}-\gamma_{18}\gamma_{11}\gamma_{16}-\gamma_{17}\gamma_{15}\gamma_{13}+\gamma_{11}\gamma_{15}\gamma_{19}+\gamma_{12}\gamma_{17}\gamma_{16}}U_{\mathrm{p}}, \tag{39}$$

$$A_{12}=\frac{\gamma_{11}\gamma_{16}+\gamma_{17}\gamma_{16}-\gamma_{14}\gamma_{19}-\gamma_{14}\gamma_{13}}{\gamma_{14}\gamma_{18}\gamma_{13}-\gamma_{14}\gamma_{12}\gamma_{19}-\gamma_{18}\gamma_{11}\gamma_{16}-\gamma_{17}\gamma_{15}\gamma_{13}+\gamma_{11}\gamma_{15}\gamma_{19}+\gamma_{12}\gamma_{17}\gamma_{16}}U_{\mathrm{p}}, \tag{40}$$

$$A_{13}=\frac{\gamma_{14}\gamma_{12}+\gamma_{14}\gamma_{18}-\gamma_{17}\gamma_{15}-\gamma_{11}\gamma_{15}}{\gamma_{14}\gamma_{18}\gamma_{13}-\gamma_{14}\gamma_{12}\gamma_{19}-\gamma_{18}\gamma_{11}\gamma_{16}-\gamma_{17}\gamma_{15}\gamma_{13}+\gamma_{11}\gamma_{15}\gamma_{19}+\gamma_{12}\gamma_{17}\gamma_{16}}U_{\mathrm{p}}。 \tag{41}$$

式中，$\gamma_{11}=\left[\mathrm{K}_1(\beta_{11}r_1)\right]'$，$\gamma_{12}=\left[\mathrm{K}_1(\beta_{12}r_1)\right]'$，$\gamma_{13}=\frac{1}{r_1}\mathrm{K}_1(\beta_{13}r_1)$，$\gamma_{14}=\chi_{11}\left[\mathrm{K}_1(\beta_{11}r_1)\right]'$，$\gamma_{15}=\chi_{12}\left[\mathrm{K}_1(\beta_{12}r_1)\right]'$，$\gamma_{16}=\chi_{13}\frac{1}{r_1}\mathrm{K}_1(\beta_{13}r_1)$，$\gamma_{17}=-\frac{1}{r_1}\mathrm{K}_1(\beta_{11}r_1)$，$\gamma_{18}=-\frac{1}{r_1}\mathrm{K}_1(\beta_{12}r_1)$，$\gamma_{19}=-\left[\mathrm{K}_1(\beta_{13}r_1)\right]'$，$U_{\mathrm{p}}$ 为 u_{p} 的 Laplace 变换。

桩周土体复阻抗可表示为

$$\begin{aligned}N_1&=-\int_0^{2\pi}(\sigma_{r1}\cos\theta-\tau_{r\theta1}\sin\theta)\Big|_{r=r_1}r_1\mathrm{d}\theta\\&=-\pi r_1[(\lambda_1+2G_1+\alpha_1M_1+\chi_{11}M_1)\beta_{11}^2A_{11}\mathrm{K}_1(\beta_{11}r_1)+\\&\quad(\lambda_1+2G_1+\alpha_1M_1+\chi_{12}M_1)\beta_{12}^2A_{12}\mathrm{K}_1(\beta_{12n}r_1)+\\&\quad G_1\beta_{13}^2A_{13}\mathrm{K}_1(\beta_{13}r_1)]=f_1U_{\mathrm{p}}\ 。\end{aligned} \tag{42}$$

2.2 桩芯土振动方程的求解

引入势函数 $\phi_{s2}(r,\theta,t)$，$\phi_{f2}(r,\theta,t)$，$\psi_{s2}(r,\theta,t)$ 和 $\psi_{f2}(r,\theta,t)$，使得

$$\left.\begin{aligned}u_{r2}&=\frac{\partial\phi_{s2}}{\partial r}+\frac{1}{r}\frac{\partial\psi_{s2}}{\partial\theta}\ ,\\u_{\theta2}&=\frac{1}{r}\frac{\partial\phi_{s2}}{\partial\theta}-\frac{\partial\psi_{s2}}{\partial r}\ ,\\w_{r2}&=\frac{\partial\phi_{f2}}{\partial r}+\frac{1}{r}\frac{\partial\psi_{f2}}{\partial\theta}\ ,\\w_{z2}&=\frac{1}{r}\frac{\partial\phi_{f2}}{\partial\theta}-\frac{\partial\psi_{f2}}{\partial r}\ 。\end{aligned}\right\} \tag{43}$$

将势函数代入式（5）～（8），并作 Laplace 变换可得

$$(\lambda_2+2G_2+\alpha_2^2M_2)\nabla^2\Phi_{s2}+\alpha_2M_2\nabla^2\Phi_{f2}=\rho_2s^2\Phi_{s2}+\rho_{\mathrm{f}}s^2\Phi_{f2}\ , \tag{44}$$

$$G_2\nabla^2\Psi_{s2}=\rho_2s^2\Psi_{s2}+\rho_{\mathrm{f}}s^2\Psi_{f2}\quad , \tag{45}$$

$$\alpha_2M_2\nabla^2\Phi_{s2}+M_2\nabla^2\Phi_{f2}=\rho_{\mathrm{f}}s^2\Phi_{s2}+m_2s^2\Phi_{f2}+b_2s\Phi_{f2}, \tag{46}$$

$$\rho_{\mathrm{f}}s^2\Psi_{s2}+m_2s^2\Psi_{f2}+b_2s\Psi_{f2}=0\quad , \tag{47}$$

式中，$\Phi_{s2}(r,\theta,s)$，$\Phi_{f2}(r,\theta,s)$，$\Psi_{s2}(r,\theta,s)$ 和 $\Psi_{f2}(r,\theta,s)$ 分别是 $\phi_{s2}(r,\theta,t)$，$\phi_{f2}(r,\theta,t)$，$\psi_{s2}(r,\theta,t)$ 和 $\psi_{f2}(r,\theta,t)$ 的 Laplace 变换。

同桩周土方程求解过程，可得

$$\begin{aligned}\Phi_{s2}=&[A_{21}\mathrm{K}_{m_2}(\beta_{21}r)+B_{21}\mathrm{I}_{m_2}(\beta_{21}r)](C_{21}\sin m_2\theta+D_{21}\cos m_2\theta)+\\&[A_{22}\mathrm{K}_{m_2}(\beta_{22}r)+B_{22}\mathrm{I}_{m_2}(\beta_{22}r)](C_{22}\sin m_2\theta+D_{22}\cos m_2\theta),\end{aligned} \tag{48}$$

式中，$\beta_{21}^2=\frac{h_{21}+\sqrt{h_{21}^2-4h_{22}}}{2}$，$\beta_{22}^2=\frac{h_2-\sqrt{h_{21}^2-4h_{22}}}{2}$，

$$h_{21}=\frac{(\lambda_2+2G_2+\alpha_2^2M_2)(m_2s^2+b_2s)+\rho_2s^2M_2-2\alpha_2M_2\rho_{\mathrm{f}}s^2}{(\lambda_2+2G_2)M_2},$$

$$h_{22}=\frac{\rho_2s^2(m_2s^2+b_2s)-\rho_f^2s^4}{(\lambda_2+2G_2)M_2}。$$

桩芯土 $r=0$ 处位移为有限值，可得 $A_{21}=A_{22}=0$。u_{r2} 为 θ 的偶函数，$u_{\theta2}$ 为 θ 的奇函数，因此 $C_{21}=C_{22}=0$，$m_2=1$。

势函数 Φ_{s2} 可表示为

$$\Phi_{s2}=[B_{21}\mathrm{I}_1(\beta_{21}r)+B_{22}\mathrm{I}_1(\beta_{22}r)]\cos\theta\ 。 \tag{49}$$

同理可有：

$$\Phi_{f2}=[B_{23}\mathrm{I}_1(\beta_{21}r)+B_{24}\mathrm{I}_1(\beta_{22}r)]\cos\theta\ , \tag{50}$$

$$\Psi_{s2}=B_{25}\mathrm{I}_1(\beta_{23}r)\sin\theta\quad , \tag{51}$$

$$\Psi_{f2}=B_{26}\mathrm{I}_1(\beta_{23}r)\sin\theta\quad , \tag{52}$$

式中，$\beta_{23}^2=\frac{\rho_2s^2(m_2s^2+b_2s)-\rho_{\mathrm{f}}^2s^4}{G_2(m_2s^2+b_2s)}$。

将式（51）～（54）代入式（44）～（47）中，可得到如下相关关系：

$$\left.\begin{aligned}B_{23}&=\chi_{21}B_{21}\ ,\\B_{24}&=\chi_{22}B_{22}\ ,\\B_{26}&=\chi_{23}B_{25}\ ,\end{aligned}\right\} \tag{53}$$

式中，$\chi_{21}=\frac{-\alpha_2M_2\beta_{21}^2+\rho_{\mathrm{f}}s^2}{M_2\beta_{21}^2-(m_2s^2+b_2s)}$，$\chi_{22}=\frac{-\alpha_2M_2\beta_{22}^2+\rho_fs^2}{M_2\beta_{22}^2-(m_2s^2+b_2s)}$，$\chi_{23}=\frac{-\rho_{\mathrm{f}}s^2}{m_2s^2+b_2s}$。

桩芯土土体和流体位移可表示为

$$U_{r2}=\left\{B_{21}\left[\mathrm{I}_1(\beta_{21}r)\right]'+B_{22}\left[\mathrm{I}_1(\beta_{22}r)\right]'+B_{25}\frac{1}{r}\mathrm{I}_1(\beta_{23}r)\right\}\cos\theta, \tag{54}$$

$$W_{r2}=\left\{\chi_{21}B_{21}\left[\mathrm{I}_1(\beta_{21}r)\right]'+\chi_{22}B_{22}\left[\mathrm{I}_1(\beta_{22}r)\right]'+\chi_{23}B_{25}\frac{1}{r}\mathrm{I}_1(\beta_{23}r)\right\}\cos\theta, \tag{55}$$

$$U_{\theta 2}=\left\{-B_{21}\frac{1}{r}\mathrm{I}_1(\beta_{21}r)-B_{22}\frac{1}{r}\mathrm{I}_1(\beta_{22}r)-A_{25}\left[\mathrm{I}_1(\beta_{23}r)\right]'\right\}\sin\theta, \tag{56}$$

$$W_{\theta 2}=\left\{-\chi_{21}B_{21}\frac{1}{r}\mathrm{I}_1(\beta_{21}r)-\chi_{22}B_{22}\frac{1}{r}\mathrm{I}_1(\beta_{22}r)-\chi_{23}B_{25}\left[\mathrm{I}_1(\beta_{23}r)\right]'\right\}\sin\theta。 \tag{57}$$

由桩芯土与桩接触面边界条件，可求得

$$B_{21}=\frac{\gamma_{25}\gamma_{29}+\gamma_{25}\gamma_{23}-\gamma_{22}\gamma_{26}-\gamma_{26}\gamma_{28}}{\gamma_{24}\gamma_{28}\gamma_{23}-\gamma_{24}\gamma_{22}\gamma_{29}-\gamma_{28}\gamma_{21}\gamma_{26}-\gamma_{27}\gamma_{25}\gamma_{23}+\gamma_{21}\gamma_{25}\gamma_{29}+\gamma_{22}\gamma_{27}\gamma_{26}}U_{\mathrm{p}}, \tag{58}$$

$$B_{22}=\frac{\gamma_{21}\gamma_{26}+\gamma_{27}\gamma_{26}-\gamma_{24}\gamma_{29}-\gamma_{24}\gamma_{23}}{\gamma_{24}\gamma_{28}\gamma_{23}-\gamma_{24}\gamma_{22}\gamma_{29}-\gamma_{28}\gamma_{21}\gamma_{26}-\gamma_{27}\gamma_{25}\gamma_{23}+\gamma_{21}\gamma_{25}\gamma_{29}+\gamma_{22}\gamma_{27}\gamma_{26}}U_{\mathrm{p}}, \tag{59}$$

$$B_{23}=\frac{\gamma_{24}\gamma_{22}+\gamma_{24}\gamma_{28}-\gamma_{27}\gamma_{25}-\gamma_{21}\gamma_{25}}{\gamma_{24}\gamma_{28}\gamma_{23}-\gamma_{24}\gamma_{22}\gamma_{29}-\gamma_{28}\gamma_{21}\gamma_{26}-\gamma_{27}\gamma_{25}\gamma_{23}+\gamma_{21}\gamma_{25}\gamma_{29}+\gamma_{22}\gamma_{27}\gamma_{26}}U_{\mathrm{p}}, \tag{60}$$

式中，$\gamma_{21}=\left[\mathrm{I}_1(\beta_{21}r_2)\right]'$，$\gamma_{22}=\left[\mathrm{I}_1(\beta_{22}r_2)\right]'$，$\gamma_{23}=\frac{1}{r_2}\mathrm{I}_1(\beta_{23}r_2)$，$\gamma_{24}=\chi_{21}\left[\mathrm{I}_1(\beta_{21}r_2)\right]'$，$\gamma_{25}=\chi_{22}\left[\mathrm{I}_1(\beta_{22}r_2)\right]'$，$\gamma_{26}=\chi_{23}\frac{1}{r_2}\mathrm{I}_1(\beta_{23}r_2)$，$\gamma_{27}=-\frac{1}{r_2}\mathrm{I}_1(\beta_{21}r_2)$，$\gamma_{28}=-\frac{1}{r_2}\mathrm{I}_1(\beta_{22}r_2)$，$\gamma_{29}=-\left[\mathrm{I}_1(\beta_{23}r_2)\right]'$。

桩芯土体复阻抗可表示为

$$N_2=\int_0^{2\pi}(\sigma_{r2}\cos\theta-\tau_{r\theta 2}\sin\theta)\Big|_{r=r_2}r_2\mathrm{d}\theta$$
$$=\pi r_2[(\lambda_2+2G_2+\alpha_2M_2+\chi_{21}M_2)\beta_{21}^2B_{21}\mathrm{I}_1(\beta_{21n}r_2)+(\lambda_2+2G_2+\alpha_2M_2+\chi_{22}M_2)\beta_{22n}^2B_{22}\mathrm{I}_1(\beta_{22n}r_2)+G_2\beta_{23}^2B_{23}\mathrm{I}_1(\beta_{23}r_2)]=f_2U_{\mathrm{p}}。 \tag{61}$$

2.3　桩振动方程的求解

对桩的振动方程（9）进行 Laplace 变换可得

$$E_{\mathrm{p}}I_{\mathrm{p}}\frac{\mathrm{d}^4U_{\mathrm{p}}}{\mathrm{d}z^4}+ms^2U_{\mathrm{p}}+(f_1+f_2)U_{\mathrm{p}}=0\quad。 \tag{62}$$

令 $\lambda^4=\frac{(ms^2+f_1+f_2)}{4E_{\mathrm{p}}I_{\mathrm{p}}}$，式（62）可化为

$$\frac{\mathrm{d}^4U_{\mathrm{p}}}{\mathrm{d}z^4}+4\lambda^4U_{\mathrm{p}}=0\quad。 \tag{63}$$

利用初参数法可求出式（63）的解：

$$U_{\mathrm{p}}=-U_0F_1(\lambda z)+\frac{\phi_0}{\lambda}F_2(\lambda z)-\frac{M_0}{E_{\mathrm{p}}I_{\mathrm{p}}\lambda^2}F_3(\lambda z)-\frac{Q_0}{E_{\mathrm{p}}I_{\mathrm{p}}\lambda^3}F_4(\lambda z)\quad, \tag{64}$$

式中，U_0，ϕ_0，M_0，Q_0 为管桩顶端的位移、转角、弯矩和剪力，F_1，F_2，F_3，F_4 为克雷洛夫函数。

将式（64）代入桩底边界条件，可得

$$\lambda F_3(\lambda H)M_0+F_4(\lambda H)Q_0=E_{\mathrm{p}}I_{\mathrm{p}}\lambda^2F_2(\lambda H)\phi_0-E_{\mathrm{p}}I_{\mathrm{p}}\lambda^3F_1(\lambda H)U_0\quad, \tag{65}$$

$$\lambda F_2(\lambda H)M_0+F_3(\lambda H)Q_0=E_{\mathrm{p}}I_{\mathrm{p}}\lambda^2F_1(\lambda H)\phi_0+4E_{\mathrm{p}}I\lambda^3F_4(\lambda H)U_0\quad, \tag{66}$$

解得

$$M_0=\frac{E_{\mathrm{p}}I_{\mathrm{p}}\lambda[F_2(\lambda H)F_3(\lambda H)-F_1(\lambda H)F_4(\lambda H)]}{F_3^2(\lambda H)-F_1(\lambda H)F_4(\lambda H)}\phi_0-\frac{E_{\mathrm{p}}I_{\mathrm{p}}\lambda^2[F_1(\lambda H)F_3(\lambda H)+4F_4^2(\lambda H)]}{F_3^2(\lambda H)-F_1(\lambda H)F_4(\lambda H)}U_0, \tag{67}$$

$$Q_0=\frac{E_{\mathrm{p}}I_{\mathrm{p}}\lambda^2[F_1(\lambda H)F_3(\lambda H)-F_2^2(\lambda H)]}{F_3^2(\lambda H)-F_1(\lambda H)F_4(\lambda H)}\phi_0+\frac{E_{\mathrm{p}}I_{\mathrm{p}}\lambda^3[F_1(\lambda H)F_2(\lambda H)+4F_3(\lambda H)F_4(\lambda H)]}{F_3^2(\lambda H)-F_1(\lambda H)F_4(\lambda H)}U_0。 \tag{68}$$

令 $s=\mathrm{i}\omega$，管桩的水平动力复阻抗 K_{h}、摇摆动力复阻抗 K_{r} 及水平–摇摆耦合动力复阻抗 K_{hr} 可表示为

$$K_{\mathrm{h}}(\mathrm{i}\omega)=\frac{E_{\mathrm{p}}I_{\mathrm{p}}\lambda^3[F_1(\lambda H)F_2(\lambda H)+4F_3(\lambda H)F_4(\lambda H)]}{F_3^2(\lambda H)-F_1(\lambda H)F_4(\lambda H)}, \tag{69}$$

$$K_{\mathrm{r}}(\mathrm{i}\omega)=\frac{E_{\mathrm{p}}I_{\mathrm{p}}\lambda[F_2(\lambda H)F_3(\lambda H)-F_1(\lambda H)F_4(\lambda H)]}{F_3^2(\lambda H)-F_1(\lambda H)F_4(\lambda H)}, \tag{70}$$

$$K_{\mathrm{hr}}(\mathrm{i}\omega)=\frac{E_{\mathrm{p}}I_{\mathrm{p}}\lambda^2[F_1(\lambda H)F_3(\lambda H)+4F_4^2(\lambda H)]}{F_3^2(\lambda H)-F_1(\lambda H)F_4(\lambda H)}\quad。 \tag{71}$$

3　管桩动力复阻抗特性分析

如无特殊说明，本文取计算参数如下：$H=10$ m，$r_1=0.5$ m，$r_2=0.38$ m，$E_{\mathrm{p}}=25$ GPa，$\rho_{\mathrm{p}}=2.5$ g/cm^3，$\rho_{\mathrm{s1}}=\rho_{\mathrm{s2}}=2.7$ g/cm^3，$\rho_{\mathrm{f}}=1.0$ g/cm^3，$G_1=G_2=10$ MPa，$\nu_1=\nu_2=0.3$，$K_{\mathrm{s1}}=K_{\mathrm{s2}}=36$ GPa，$K_{\mathrm{f}}=2$ GPa，$n_1=n_2=0.3$，$k_{\mathrm{d1}}=k_{\mathrm{d2}}=10^{-6}$ m/s。

3.1　合理性验证

为了验证本文解的合理性，将本文解与尚守平等[8]的实心桩解进行对比。令 $r_2\to 0$，即将 PCC 桩退化为实心桩，得到饱和土地基中实心桩水平振动解，将其与尚守平解[8]比较。图 2 表明本文解与尚守平解得到的结果基本一致。尚守平解由于忽略了土中流体惯性项以及假设土颗粒和流体都不可压缩，水平动力复阻抗的实部在高频范围内比本文解略小。

3.2　桩长对管桩动力复阻抗的影响

复阻抗的实部代表真实的刚度，虚部代表反映桩–土系统中能量消散的阻尼。复阻抗受桩长、桩内外半径等因素的影响，考虑到工程中实际感兴趣的频率范围较低，图 3～8 给出了 0～200 Hz 范围内它们的曲线关系。

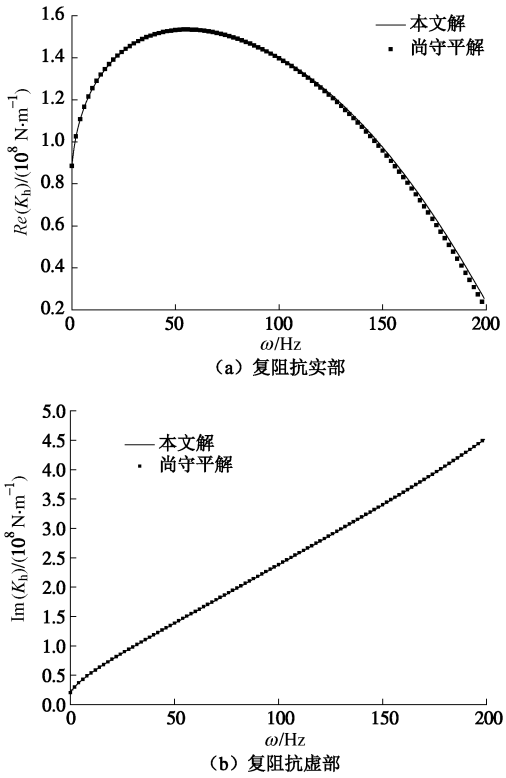


图 2 本文解与尚守平解对比

Fig. 2 Comparison between Author's and Shang's solution

图 3～5 反映了桩长对管桩动力复阻抗的影响，由图可见，随着桩长的增大，在低频范围内，3 种动力复阻抗的实部均减小；在高频范围内，3 种动力复阻抗的实部均增大。这是因为低频范围内复阻抗主要取决于管桩的静力承载力，而由于桩底固定，静承载力

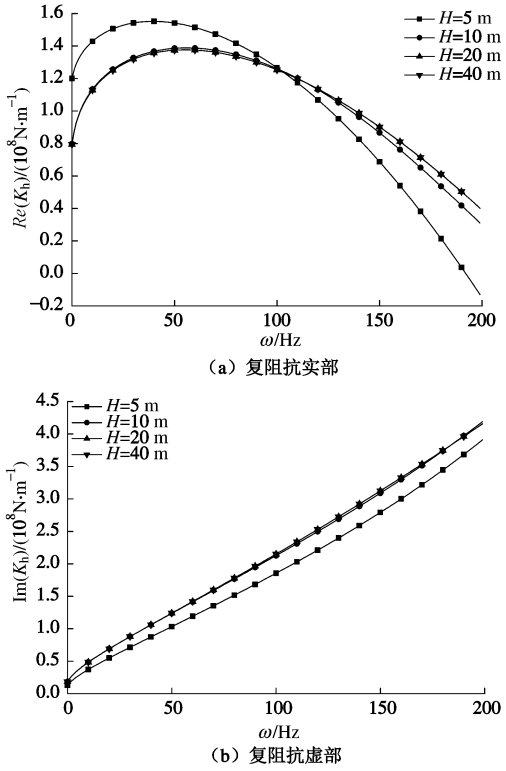


图 3 桩长对管桩水平动力复阻抗的影响

Fig. 3 Variation of horizontal complex impedance of pipe pile with pile length

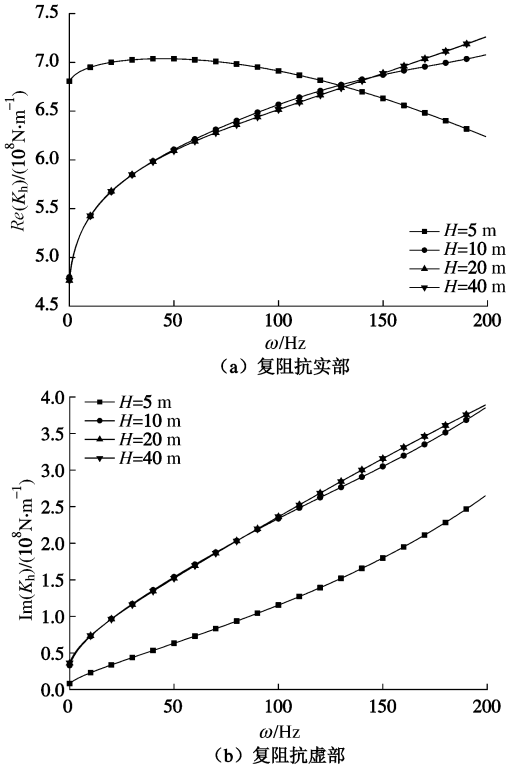


图 4 桩长对管桩摇摆动力复阻抗的影响

Fig. 4 Variation of rocking complex impedance of pipe pile with pile length

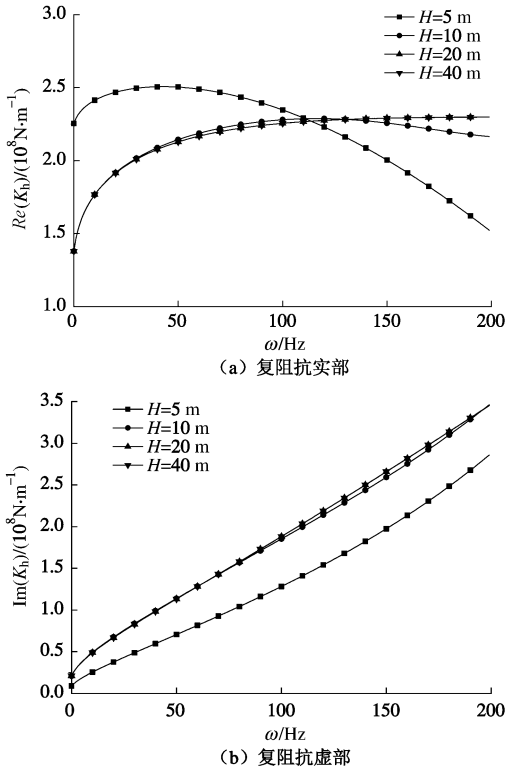


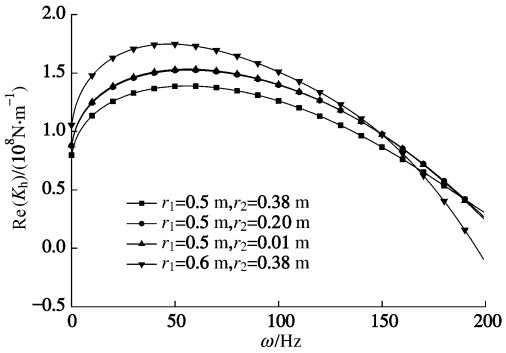
图 5 桩长对管桩水平－摇摆耦合动力复阻抗的影响

Fig. 5 Variation of horizontal-rocking complex impedance of pipe pile with pile length

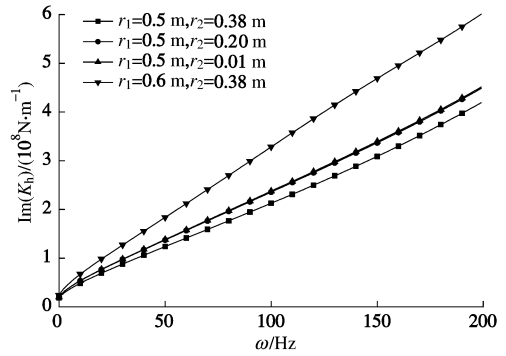
随着桩长细比增大而减小；高频范围内桩–土耦合振动逐渐发挥作用，桩长越长桩–土耦合作用越大，因此复阻抗增大。在全部频率范围内，3 种动力复阻抗的虚部均随着桩长的增大而增大。当桩长大于 20 m 时，继续增加桩长对复阻抗影响不大。

3.3　桩半径对管桩复阻抗的影响

图 6～8 给出了桩半径对管桩动力复阻抗的影响，从图可见，随着内半径的减小，3 种动力复阻抗均增大。$r_2=0$ 与 $r_2=0.2$ m 时，动力复阻抗基本相同，说明当 $r_2<0.2$ m 时，内半径影响不明显。随着外半径的增大，3 种动力复阻抗均增大。$r_2=0.6$ m，$r_2=0.38$ m 的管桩复阻抗明显大于 $r_1=0.5$ m，$r_2=0.2$ m 的管桩复阻抗，而两者的截面积基本相等。这表明，管桩的动力复阻抗随着平均半径的增大而增大。



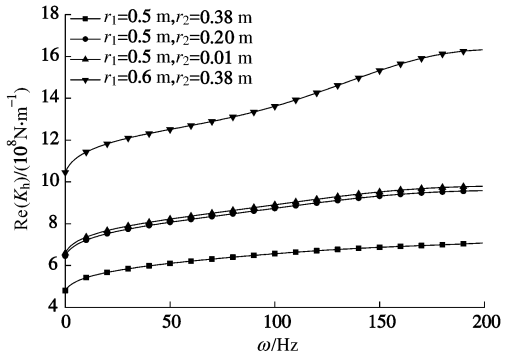
（a）复阻抗实部



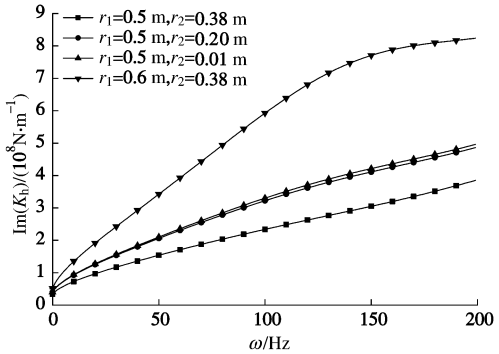
（b）复阻抗虚部

图 6　桩半径对管桩水平动力复阻抗的影响

Fig. 6 Variation of horizontal complex impedance of pipe pile with pile radii



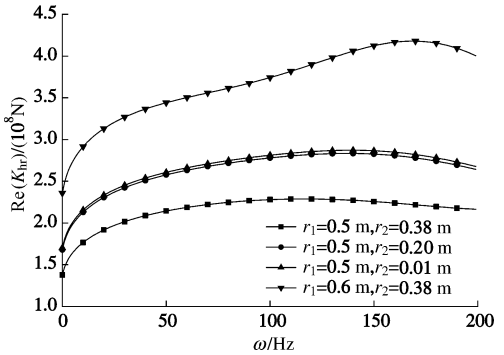
（a）复阻抗实部



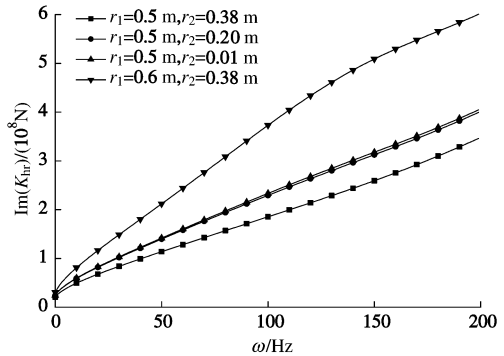
（b）复阻抗虚部

图 7　桩半径对管桩摇摆动力复阻抗的影响

Fig. 7 Variation of rocking complex impedance of pipe pile with pile radii



（a）复阻抗实部



（b）复阻抗虚部

图 8　桩半径对管桩水平–摇摆动力复阻抗的影响

Fig. 8 Variation of horizontal-rocking complex impedance of pipe pile with pile radii

4　结　　语

本文基于经典的Novak平面应变假定，推导了饱和土地基中PCC桩水平振动响应解析解，给出了大直径管桩动力复阻抗的解析表达式，为PCC桩动力特性研究及应用提供了新的理论支持。研究结果表明：随着桩长的增大，在低频范围内，复阻抗的实部均减小；在高频范围内，复阻抗的实部均增大。在全部频率范围内，阻抗的虚部均随着桩长的增大而增大。当桩长>20 m时，继续增加桩长对复阻抗影响不大。随着内半径的减小，复阻抗的实部和虚部均增大。但当 $r_2<0.2$ m 时，内半径影响不明显。随着外半径的增

大，复阻抗的实部和虚部均增大。等截面积的管桩，动力复阻抗随着平均半径的增大而增大。

参考文献：

[1] NOGAMI T, KONAGAI K. Time domain flexural response of dynamically loaded single piles[J]. Journal of Engineering Mechanics, ASCE, 1986, **114**(9): 1512 – 1525.

[2] MARKIS N, GAZETAS G. Displacement phase differences in a harmonically oscillating pile[J]. Géotechnique, 1993, **43**(1): 135 – 150.

[3] YAO S, NOGAMI T. Lateral cyclic response of pile in viscoelastic Winkler subgrade[J]. Journal of Engineering Mechanics, 1994, **120**(4): 785 – 775.

[4] NOVAK M, ABOUL-ELLA F. Dynamic stiffness and damping of piles[J]. Canadian Geotechnical Journal, 1974, **11**(4): 575 – 598.

[5] NOVAK M, NOGAMI T, ABOUL-ELLA F. Dynamics soil reactions for plane strain case[J]. Journal of the Engineering Mechanics Division, ASCE, 1978, **104**(4): 953 – 959.

[6] GAZETAS G, DOBRY R. Horizontal response of piles in layered soils[J]. Journal of Geotechnical Engineering, ASCE, 1984, **110**(1): 20 – 40.

[7] 杨 军, 宋二祥, 陈肇元. 桩在饱和土中水平振动的辐射阻尼简化算法[J]. 岩土力学, 2002, **23**(2): 179 – 183. (YANG Jun, SONG Er-xiang, CHEN Zhao-yuan. Simple radiation damping model for horizontally vibrating pile in saturated soil[J]. Rock and Soil Mechanics, 2002, **23**(2): 179 – 183. (in Chinese))

[8] 尚守平, 余 俊, 王海东, 等. 饱和土中桩水平振动分析[J]. 岩土工程学报, 2007, **29**(11): 1696 – 1702. (SHANG Shou-ping, YU Jun, WANG Hai-dong, et al. Horizontal vibration of piles in saturated soil[J]. Chinese Journal of Geotechnical Engineering, 2007, **29**(11): 1696 – 1702. (in Chinese))

[9] 刘汉龙, 费 康, 马晓辉, 等. 振动沉模大直径现浇薄壁管桩技术及其应用(Ⅰ): 开发研制与设计[J]. 岩土力学, 2003, **24**(2): 164 – 168. (LIU Han-long, FEI Kang, MA Xiao-hui, et al. Cast-in-situ concrete thin-wall pipe pile with vibrated and steel tube mould technology and its application: I development and design[J]. Rock and Soil Mechanics, 2003, **24**(2): 164 – 168. (in Chinese))

[10] 刘汉龙, 郝小员, 费 康, 等. 振动沉模大直径现浇薄壁管桩技术及其应用(Ⅱ)工程应用与现场试验[J]. 岩土力学, 2003, **24**(3): 372 – 375. (LIU Han-long, HAO Xiao-yuan, FEI Kang, et al. Field pour concrete thin wall cased pile technology and its application (Ⅱ): application and in-situ test[J]. Rock and Soil Mechanics, 2003, **24**(3): 372 – 375. (in Chinese))

[11] DING X M, LIU H L, LIU J Y, et al. Wave propagation in a pipe pile for low-strain integrity testing[J]. Journal of Engineering Mechanics, ASCE, 2011, **117**(9): 598 – 609.

[12] DING X M, LIU H L, ZHANG B. High-frequency interference in low strain integrity testing of large-diameter pipe piles[J]. Science China (Technological Sciences), 2011, **54**(2): 420 – 430.

[13] 丁选明, 刘汉龙. 轴对称均匀黏弹性地基中现浇薄壁管桩纵向动力响应简化解析方法[J]. 岩土力学, 2008, **29**(12): 3353 – 3359. (DING Xuan-ming, LIU Han-long. Simplified analytical method of vertical dynamic response of cast-in-situ concrete thin-wall pipe piles in axisymmetric homogeneous viscoelastic soil[J]. Rock and Soil Mechanics, 2008, **29**(12): 3353 – 3359. (in Chinese))

[14] 丁选明, 刘汉龙. 大直径管桩在瞬态集中荷载作用下的振动响应时域解析解[J]. 岩土工程学报, 2013, **35**(6): 1010 – 1017. (DING Xuan-ming, LIU Han-long. Time-domain analytical solution of the vibration response of a large-diameter pipe pile due to transient concentrated load[J]. Chinese Journal of Geotechnical Engineering, 2013, **35**(6): 1010 – 1017. (in Chinese))

[15] 郑长杰, 丁选明, 刘汉龙, 等. 考虑土体三维波动效应的现浇大直径管桩纵向振动[J]. 岩土工程学报, **35**(12): 2247 – 2254. (ZHENG Chang-jie, DING Xuan-ming, LIU Han-long, et al. Analytical solution to vertical vibration of cast-in-place concrete large-diameter pipe piles by considering 3D wave effect of soils[J]. Chinese Journal of Geotechnical Engineering, **35**(12): 2247 – 2254. (in Chinese))

[16] BIOT M A. The theory of propagation of elastic waves in a fluid-saturated porous soild: I low-frequency range[J]. The Journal of the Acoustical Society of America, 1956, **28**: 168 – 178.

[17] SENJUNTICHAI T, RAJAPAKSE R K N D. Transient response of a circular cavity in a poroelastic medium[J]. International Journal for Numerical and Analytical Methods in Geomechanics, 1993, **17**(6): 357 – 383.